Set Theory

Definition –

Set – A well-defined collection of distinct object is called set.

- objects in the set are element.
- we denote set by A, B, C, X, Y, Z ---
- and element by a, b, c, x, y, z ---
- Ex – $A = \{x, y, z\}$

$N = \{1, 2, 3, ---\}$

Representation –

① **Roster or Tabular form** –

all the element listed within { } and separated by ,

Ex ① B = set of all even +ve integers less than 10

or

$B = \{2, 4, 6, 8\}$

Ex 2 A = set of all factors of 12

or

$A = \{1, 2, 3, 4, 6, 12\}$

- (2) <u>Rull Method or Set Builder form</u>:
- in this we list the property statisfied by all the ~~se~~ element of set, it is written as

$$\boxed{\{x ; x \text{ satisfy the property}\}}$$

<u>Ex-1</u> A = { set of all natural numbers less than 9}

or

$A = \{x ; x \in N \text{ and } x < 9\}$

<u>Ex-2</u> complex number

$C = \{x + iy ; x, y \in R\}$

<u>Ex-3</u> $Q = \{\frac{p}{q} ; p, q \in Z, q \neq 0\}$

Types of Sets –

① * **Empty Set or Null set or void set**

A set containing no element at all is called empty set.

– it is denoted by $\boxed{\phi \text{ or } \{\ \}}$

Ex (1) $A = \{x ; x \in N, \ \& \ 2 < x < 3\} = \phi$

(2) $\{x ; x^2 = 9 \ \&$ x is an even integer $\} = \phi$

Note – if a set has at least one element is called non-empty set

② **Singleton set** – A set containing only one element

Ex (i) $\{x ; x \in N$ and $x^2 - 9 = 0\} = \{3\}$

(ii) $\{0\}$

Note

Cardinality of a set or cardinal number

- No. of distinct element contain in a set A
- it is denoted by $\boxed{n(A) \text{ or } |A|}$
- Ex $X = \{x, y, z, t\}$

$$n(X) = 4$$

(3) Finite and infinite set —

When any set have finite No. of element is finite set otherwise infinite set

Ex (i) $A = \{2, 4, 6, 8, 10\}$ → Finite

(ii) the set of all person on the earth, is finite set

Infinite set

$$N = \{1, 2, 3, ---\}$$

$$I = \{---, -3, -2, -1, 0, 1, 2, 3, ---\}$$

Equal set — two set A and B are equal if they have same No. of element

$\boxed{A = B}$

Ex (i) $A = \{1, 2, 3\}$
$B = \{3, 2, 1\}$ $\rightarrow$ $\boxed{A = B}$

(ii) $A = \{1, 2, 3\}$
$B = \{1, 2, 3, 3\}$ $\rightarrow$ $\boxed{A = B}$

$\Downarrow$

$B = \{1, 2, 3\}$

⑤ Equivalent sets

Two set A and B are equivalent if $\underbrace{n(A) = n(B)}_{\text{cardinality}}$

Ex $A = \{2, 3, 4\} \Rightarrow n(A) = 3$
$B = \{7, 8, 9\} \Rightarrow n(B) = 3$

$\boxed{n(A) = n(B) = 3}$

So A and B are equivalent set.

Power set – The set of all subset of a given set A is called power set of A. and denoted by P(A).

Note – $\boxed{\text{If } n(A) = m \text{ then } n[P(A)] = 2^m}$

Ex – write down all possible subset of

(i) $A = \{2, 3\}$

ii $B = \{a, b\}$

(iii) $C = \{a, b, c\}$

Soln (i) $A = \{2, 3\}$

$P(A) = \{\phi, \{2\}, \{3\}, \{2, 3\}\}$

$\because n(A) = 2$ so $n[P(A)] = 2^2$
$= 4$

(iii) $C = \{a, b, c\}$

$P(C) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

$\because n(C) = 2^{3}$

$\therefore n[P(A)] = 2^3 = 8$

Ex Two finite set have m and n elements, The total no. of subsets of 1st set 56 more than total no. of subsets of second set.
Find the value of m and n ?

Sol^n Let set A have m element
B have n element

i.e. $n(A) = m$, $n(B) = n$ —(I)

if a set A has m elements then total no. of subset A is 2^m.

$$\left.\begin{aligned} n[P(A)] &= 2^m \\ \& \quad n[P(B)] &= 2^n \end{aligned}\right\} \longrightarrow (II)$$

given $n[P(A)] = n[P(B)] + 56$

$$2^m = 2^n + 56$$

$$2^m - 2^n = 56$$

$$2^n[2^{m-n} - 1] = 2^3 \times 7 \quad \text{—(III)}$$

compare

$$2^n = 2^3 \Rightarrow \boxed{n=3}$$

$$2^{m-n} - 1 = 7$$

$$2^{m-n} = 8 = 2^3$$

$$2^{m-3} = 2^3$$

(2) $m - 3 = 3$, $\boxed{m = 6}$

Operations of sets

Natural No. →

$N = \{1, 2, 3, \ldots\}$

Whole No →

$W = \{0, 1, 2, 3, \ldots\}$

Integer (Z or I) →

$Z = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\underbrace{-3, -2, -1}_{-ve\ int}$ $\quad$ $\underbrace{1, 2, 3, \ldots}_{+ve\ int}$

$\underbrace{0, 1, 2, 3, \ldots}_{No\ -ve\ int}$

Rational No. → (Q)

$Q = \{\frac{p}{q} : p, q \in Z, q \neq 0\}$

Note - all the integers are Rational No.

$\{\ldots -\frac{3}{1}, -\frac{2}{1}, -\frac{1}{1}, \frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \ldots\}$

condition satisfied

Irrational No. → (Qi) this is $\frac{p}{q}$ type element

condition Result of $\frac{p}{q}$ is non-terminating and non-repeatative.

Ex $\frac{22}{7} \Rightarrow$

$$
\begin{array}{r}
7\overline{)22}(3.142\ldots \\
\underline{21} \\
10 \\
\underline{7} \\
30 \\
\underline{28} \\
20 \\
\underline{14} \\
6
\end{array}
$$

so $\boxed{Q_i = \{\pi, \sqrt{2}, \sqrt{3} \ldots\}}$

Real No. $\rightarrow$ (R) $\rightarrow$

$\boxed{R = Q \cup Q_i}$

$R = \{\ldots -4, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\downarrow$

(1, 2) infinite value

Note $\boxed{N C W C I C Q C R}$

complex No. $\rightarrow$ $i^2 = -1$

$C = \{x + iy : x, y \in R\}$

$N C W C I C Q C R C$ complex No. (C)

Operations on the Set

① **Union of Sets** – $(A \cup B)$ is the set of all those elements which are either in A or in B or both A and B.

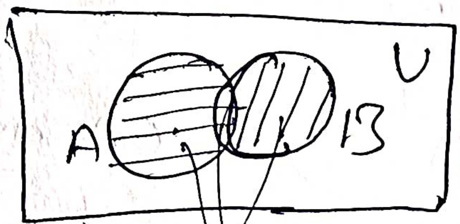
$$\boxed{A \cup B = \{x : x \in A \text{ or } x \in B\}}$$

Note – (i) $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

(ii) $x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$

Ex-1 $A = \{2, 3, 4\}$ $B = \{1, 5, 4, 6\}$

Thus $A \cup B = \{1, 2, 3, 4, 5, 6\}$



$A \cup B$

Ex-2 $A = \{x : x$ is a prime number less than 10$\}$

$B = \{x : x \in N, x$ is the factor of 8$\}$

Find $(A \cup B) = ?$

Solⁿ – $A = \{2, 3, 5, 7\}$

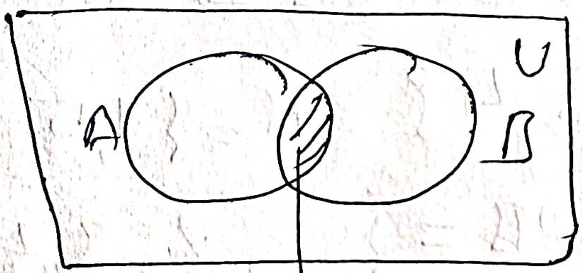
$B = \{1, 2, 4, 8\}$

$A \cup B = \{1, 2, 3, 4, 5, 7, 8\}$

(2) **Intersection of two sets** –

$(A \cap B)$ is the set of all those elements which are common to both set A and B.

$$\boxed{(A \cap B) = \{x : x \in A \text{ and } x \in B\}}$$



$A \cap B$

Note – (i) $x \in (A \cap B) \Rightarrow x \in A$ and $x \in B$

(ii) $x \notin (A \cap B) \Rightarrow x \notin A$ or $x \notin B$

Ex. $A = \{1, 3, 5, 7, 9\}$

$B = \{2, 3, 5, 7, 11, 13\}$

$A \cap B = \{3, 5, 7\}$

Ex 2 if $A_i = \{0, i\}$ where $i \in Z$ the set of integer

(i) $A_1 \cup A_2$ (ii) $A_3 \cap A_4$

(iii) $\bigcup_{i=5}^{10} A_i$

Sol^n- $A_i = \{0, i\}$ $A_1 = \{0, 1\}$ $A_2 = \{0, 2\}$

$A_3 = \{0, 3\}$ $A_4 = \{0, 4\}$

(i) $A_1 \cup A_2 = \{0, 1, 2\}$

(ii) $A_3 \cap A_4 = \{0\}$ Singleton set

(iii) $\bigcup_{i=5}^{10} A_i \Rightarrow A_5 \cup A_6 \cup A_7 \cup A_8 \cup A_9 \cup A_{10}$

$\Rightarrow$ $i=5$ A_5

$i=6$ A_6

$\vdots$ $\vdots$

$i=10$ A_{10}

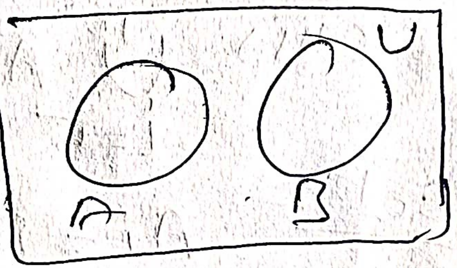
$\Rightarrow \{0, 5\} \cup \{0, 6\} \cup \{0, 7\} \cup \{0, 8\} \cup \{0, 9\} \cup \{0, 10\}$

$\Rightarrow \{0, 5, 6, 7, 8, 9, 10\}$

Disjoint set —

two set A and B are said to be disjoint if no any element are common.

$$\boxed{A \cap B = \phi}$$



$A = \{1, 2, 3\}$

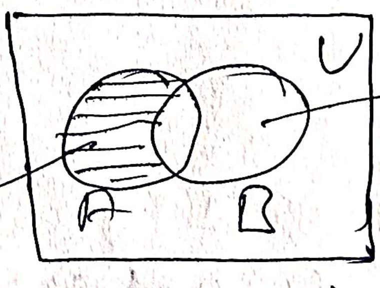
$B = \{5, 6, 7\}$

$A \cap B = \phi$

Difference of Sets —

for any set A and B, their diffrence $(A - B)$ is defined as

$$\boxed{A - B = \{x ;\ x \in A \text{ and } x \notin B\}}$$



B − A

A − B

Note— $x \in A - B \Rightarrow x \in A$ and $x \notin B$

Ex-1 $A = \{1, 2, 3, 4, 5\}$

$B = \{5, 6, 7\}$

$A - B = \{1, 2, 3, 4\}$

Ex-2

If $A = \{x; x \in N, x \text{ is a factor of } 6\}$

$B = \{x; x \in N, x \text{ is a factor of } 8\}$

$A - B = ?$ $B - A = ?$

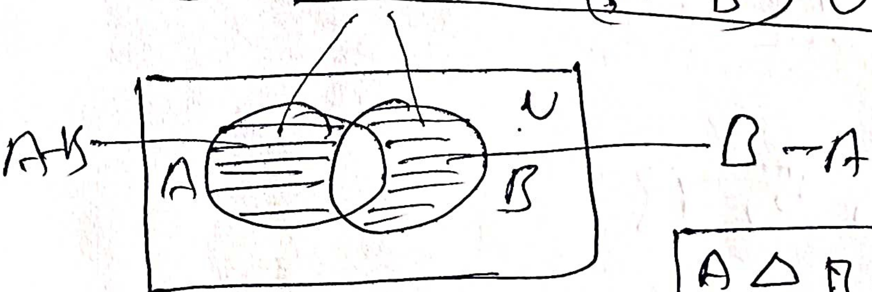
Soln —

$A = \{1, 2, 3, 6\}$

$B = \{1, 2, 4, 8\}$

$A - B = \{3, 6\}$ $B - A = \{4, 8\}$

Symmetric difference — of two set A and B is denoted by $A \triangle B$ and defined by $\boxed{A \triangle B = (A - B) \cup (B - A)}$



or

$\boxed{A \triangle B = (A \cup B) - (A \cap B)}$

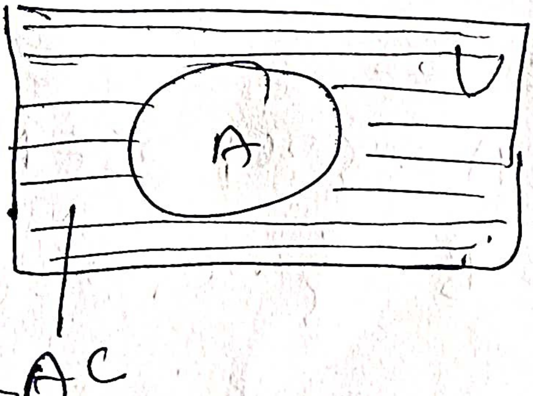
from the above Example

$A \triangle B = \{3, 4, 6, 8\}$

complement of a set —

Let U is Universal set and let $A \subseteq U$. Then complement of A denoted by A' or A^c and defined as

$\boxed{A' \text{ or } A^c = U - A}$



$\boxed{A^c = \{x \in U : x \notin A\}}$

$\boxed{x \in A^c \Rightarrow x \notin A}$

Note – (i) $A \cap A^c = \phi$

(ii) $A \cup A^c = U$

Ex $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A = \{2, 4, 6, 8\}$

(i) A' (ii) $(A')'$

Soln (i) $A^c = U - A$

$= \{1, 3, 5, 7\}$

(ii) $(A^c)^c = U - A^c$

$= \{2, 4, 6, 8\} = A$